## Math 241 Sample Problems for Exam 2

Question 1 Use Lagrange multipliers to find the maximum and minimum values of f(x, y, z) = x - 2y + 5z on the sphere  $x^2 + y^2 + z^2 = 30$ .

Question 2 Evaluate the following double integral:

$$\int_0^2 \int_{y/2}^1 y e^{x^3} \, dx \, dy$$

**Question 3** Find the volume of the solid in space which lies below the surface  $z = 3 + \cos y$  and above the region in the xy-plane bounded by the curves  $x = \pi$ , y = 0, and y = 2x by evaluating an appropriate double integral.

**Question 4** Let R be the solid region bounded by the planes x = 0, y = 0, z = 2, and the paraboloid  $z = x^2 + y^2$ , in the first octant. Compute  $\iiint_R x \, dV$ .

**Question 5** Find the volume determined by  $z \le 6 - x^2 - y^2$  and  $z \ge \sqrt{x^2 + y^2}$ .

**Question 6** Convert the integral  $\int_{-\sqrt{2}}^{\sqrt{2}} \int_{-\sqrt{2-y^2}}^{\sqrt{2-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{4-x^2-y^2}} z^2 dz dx dy$  to spherical coordinates. Don't evaluate it.

**Question 7** Evaluate  $\iint_R \exp\left(\frac{y-x}{y+x}\right) dA$  where *R* is the triangle with vertices (0,0), (1,0), (0,1), by using the change of variables  $x = \frac{1}{2}(v-u)$ ,  $y = \frac{1}{2}(u+v)$ .

## Question 8

a) Evaluate the line integral  $\int_{\mathbf{c}} x^2 y \, dx + xy^3 \, dy$ , where **c** consists of the line segments from (0,0) to (3,3) and from (3,3) to (0,3).

b) Evaluate the line integral  $\int_{\mathbf{c}} (e^y + ye^x) dx + (e^x + xe^y) dy$ , where **c** is the part of the graph  $y = \ln 6x$  joining (1/6, 0) to  $(1/2, \ln 3)$ .

**Question 9** Find the area of the surface that is part of the sphere  $x^2 + y^2 + z^2 = 4z$  that lies inside the paraboloid  $z = x^2 + y^2$ .